

## Honors Final Exam Review Table of Contents

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*Note: Each topic contains a problem set.*

## FINAL EXAM REFERENCE SHEET

**Equation of a Circle:**  $(x - h)^2 + (y - k)^2 = r^2$

**Area of a Circle:**  $\pi r^2$

**Circumference:**  $2\pi r$

**Trigonometric Functions:**  $\sin \theta = \frac{opp}{hyp}$        $\cos \theta = \frac{adj}{hyp}$        $\tan \theta = \frac{opp}{adj}$

**Distance formula:**  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

**Volume<sub>cylinder</sub>:**  $\pi r^2 h$

**Volume<sub>cone</sub>:**  $\frac{1}{3} \pi r^2 h$

**Volume<sub>sphere</sub>:**  $\frac{4}{3} \pi r^3$

**Volume<sub>prism</sub>:**  $Bh$  (B = area of the base)

$m^\circ \angle A$	$\sin A$	$\cos A$	$\tan A$	$m^\circ \angle A$	$\sin A$	$\cos A$	$\tan A$
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.50	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.50	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1	90	1	0	Undefined



Name: \_\_\_\_\_





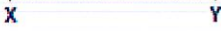

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### FINAL EXAM REVIEW – Basic Geometry Vocabulary

Euclid:

- the father of Geometry
- studied by Abraham Lincoln
- built an *axiomatic* system of Geometry
  - based on **axioms** – statements accepted as true
  - ex: A straight line segment can be drawn joining any two points.

	Description	Figure	Symbol
<b>point*</b>	describes a location; zero dimensions		P or Point P
<b>line*</b>	a collection of points along a straight path with no endpoints; one dimension (length)		$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
<b>plane*</b>	a flat surfaces that extends indefinitely; two dimensions (length and width)		Plane EFG or Plane $\psi$
<b>ray</b>	a collection of points along a straight path with one endpoint which extends indefinitely in one direction; one dimension (length)		$\overrightarrow{PQ}$
<b>line segment</b>	a collection of points along a straight path with two endpoints; one dimension (length) *measurable		$\overline{XY}$ or $\overline{YX}$
<b>angle</b>	two rays that meet at a point (this point is the <b>vertex</b> ) *measurable		$\angle ABC$

\***undefined** terms of Geometry... MANY definitions have their roots in these three words

Directions: I HIGHLY recommend answering these on a separate sheet of paper.

segment, ray PA, line PQ, ray QP

1. Describe what each of these symbols means:  $\overline{PQ}$ ,  $\overrightarrow{PQ}$ ,  $\overleftrightarrow{PQ}$ ,  $\overrightarrow{QP}$ .

2. Sketch a line that contains point  $R$  between points  $S$  and  $T$ . Which of the following are true?



A.  $\overrightarrow{SR}$  is the same as  $\overrightarrow{ST}$ .  $\checkmark$

B.  $\overrightarrow{SR}$  is the same as  $\overrightarrow{RT}$ .  $\checkmark$

C.  $\overrightarrow{RS}$  is the same as  $\overrightarrow{TS}$ .  $\checkmark$

D.  $\overrightarrow{RS}$  and  $\overrightarrow{RT}$  are opposite rays.  $\checkmark$

E.  $\overline{ST}$  is the same as  $\overline{TS}$ .  $\checkmark$

F.  $\overrightarrow{ST}$  is the same as  $\overrightarrow{TS}$ .  $\checkmark$

Decide whether the statement is true or false.

3. Points  $A$ ,  $B$ , and  $C$  are collinear.  $\checkmark$

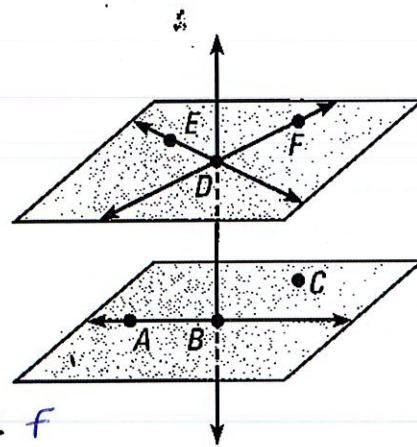
4. Points  $A$ ,  $B$ , and  $C$  are coplanar.  $\checkmark$

5. Point  $F$  lies on  $\overleftrightarrow{DE}$ .  $\checkmark$

6.  $\overleftrightarrow{DE}$  lies on plane  $DEF$ .  $\checkmark$

7.  $\overleftrightarrow{BD}$  and  $\overleftrightarrow{DE}$  intersect.  $\checkmark$

8.  $\overleftrightarrow{BD}$  is the intersection of plane  $ABC$  and plane  $DEF$ .  $\checkmark$



Short Answer:

1. What are the undefined terms?

Point, line, plane

2. Why will two points ALWAYS be collinear? Why will three points always be coplanar?

① B/c there is always a unique line that will connect any two points

② B/c for any 3 points, there exists a plane that contains all 3 points

3. In what way(s) was Euclid influential?

- built/organized the system of Geometry based on a set of axioms (statements accepted as true w/o proof)
- Lincoln studied Euclid to build his skills in logic to use toward making a good, structured argument



# FINAL EXAM REVIEW – Constructions

**Perpendicular Bisector:** <http://www.mathopenref.com/constbisectline.html>

- Given line segment AB
- Draw circle A (this means the center is A) with radius AB OR Draw circle A with radius of over half of AB
- Draw circle B (this means the center is B) with the same radius
- Label the two intersection points C and D
- Draw a line between C and D

**Angle Bisector:** <http://www.mathopenref.com/constbisectangle.html>

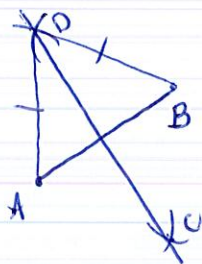
- Set the compass to any width
- Draw an arc with center A (vertex) (or draw a circle with center A) so that it intersects with both sides of the angle
- Label the intersection points as B and C (these are the points where the arc intersected the angle)
- Draw an arc with center B/draw a circle with center B
- Draw an arc with center C with the compass set to the same width (set to the same radius)
- Label the intersection point D
- Draw a ray/line/line segment from A to D

**Equilateral Triangle:** <http://www.mathopenref.com/constequilateral.html>

- Given line segment AB
- Draw circle A with radius AB
- Draw circle B with radius AB
- Mark one intersection point of the circles as C
- Create line segments AC and BC

## WHY DO THEY WORK?

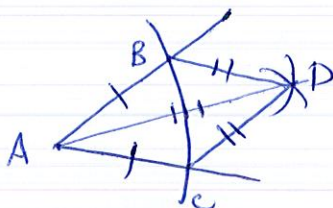
Perpendicular Bisector



•  $\overline{AD} \cong \overline{BD}$  b/c they are radii of  $\cong$  circles

• all pts. on a  $\perp$  bisector are equidistant from the endpoints of the segment being bisected

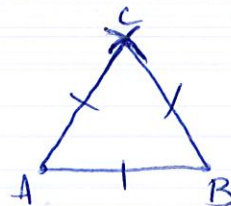
Angle Bisector



•  $\overline{AB} \cong \overline{AC} \rightarrow$  radii of the same circle  
•  $\overline{BD} \cong \overline{BC} \rightarrow$  radii of  $\cong$  circles

•  $\overline{AD} \cong \overline{AD} \rightarrow$  reflexive prop.  
•  $\triangle ABD \cong \triangle ACD \rightarrow$  SSS  
•  $\angle BAD \cong \angle CAD \rightarrow$  CPCTC

Equilateral  $\triangle$



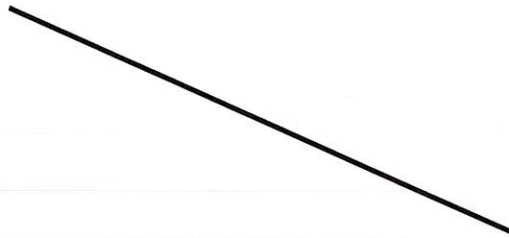
•  $\overline{AC} \cong \overline{AB} \cong \overline{CB} \rightarrow$  radii of  $\cong$  circles

## WLPCS

### Geometry

You can do these on a separate sheet of paper by drawing a line segment with a straight edge.

1. Construct the perpendicular bisector of  $\overline{DE}$ . When complete, mark the congruent segments and right angles.



2. Construct an equilateral triangle.



3. How does the construction of an equilateral triangle ensure that all sides are congruent?

*The sides are radii of congruent circles.*

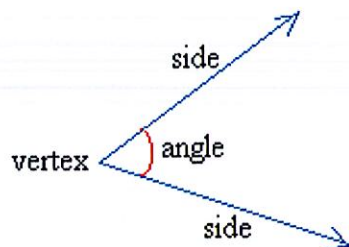
4. How does the construction of an angle bisector ensure that two congruent angles are created?

*By creating 2 congruent triangles (SSS), the angles are  $\cong$  by CPCTC*

**\*\* a similar question regarding perpendicular bisectors will be later in the review \*\***

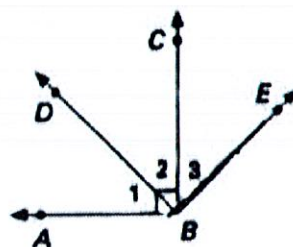
FINAL EXAM REVIEW – Angle Relationships and Angle Proofs

What is an angle?



\*the angle is the space between the two sides... a portion of  $360^\circ$

How do you name an angle?



$\angle 1$  can also be named  $\angle ABD$  or  $\angle DBA$

$\angle 2$  can also be named  $\angle DBC$  or  $\angle CBD$

**\*notice that the VERTEX is always the letter in the middle\***

How can you describe an angle?

*Acute angle*

less than  $90^\circ$



*Right angle*

=  $90^\circ$



*Obtuse angle*

between  $90^\circ$   
and  $180^\circ$



*Straight line*

=  $180^\circ$



*Reflex angle*

greater than  
 $180^\circ$



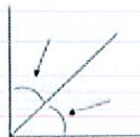
Angle relationships



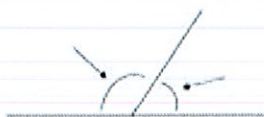
adjacent angles



vertical angles



complementary angles

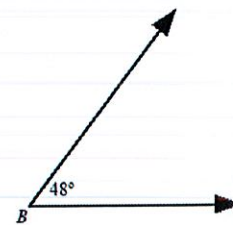
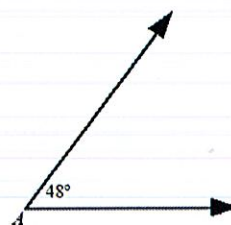


supplementary angles

Congruent Angles

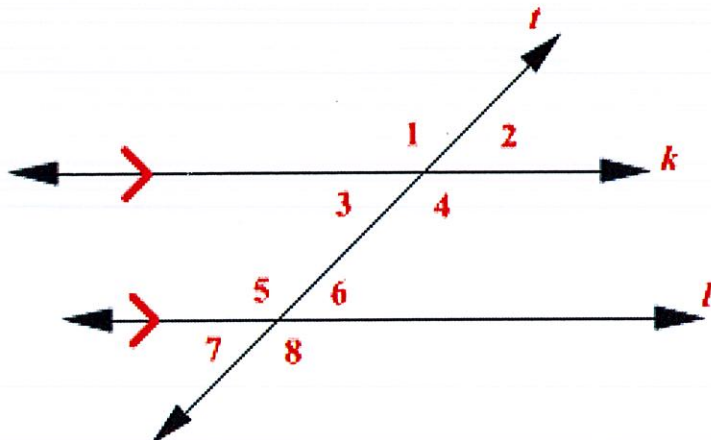
Congruent angles have the same angle measure.

$\angle A \cong \angle B$  because the measure of both angles is  $48^\circ$ .





### Parallel Lines cut by a Transversal



In the diagram to the left, line  $l$  and line  $k$  are parallel. Line  $t$  is a **transversal** because it cuts through both lines.

#### Key Points:

1. Vertical angles are congruent.

Ex:  $\angle 1 \cong \angle 4$  and  $\angle 7 \cong \angle 6$

2. Alternate interior angles are congruent.

Ex:  $\angle 3 \cong \angle 6$  and  $\angle 5 \cong \angle 4$

3. Corresponding angles are congruent.

Ex:  $\angle 1 \cong \angle 5$  and  $\angle 8 \cong \angle 4$

4. Same side interior angles are supplementary.

Ex:  $m\angle 6 + m\angle 4 = 180^\circ$

5. Same side exterior angles are supplementary.

Ex:  $m\angle 8 + m\angle 2 = 180^\circ$

6. Linear pairs are supplementary.

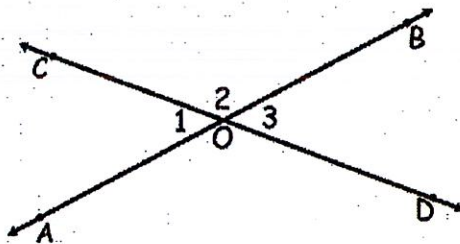
Ex:  $m\angle 1 + m\angle 2 = 180^\circ$

- How can we prove lines are parallel?
  - If alternate interior, alternate exterior, or corresponding angles are congruent.
  - If same side interior or same side exterior angles are supplementary.
- If we are given that lines are parallel, we can state that We can state that alternate interior or exterior angles and corresponding angles are congruent and same side interior or exterior angles are supplementary if the lines are parallel.
- Corresponding Angles Postulate: When parallel lines are cut by a transversal, corresponding angles are congruent. This is a **postulate** – something accepted as true without proof – so this is **very** helpful to use in proofs.
- Corresponding Angles CONVERSE: When corresponding angles are congruent, the lines are parallel (*simplified version*).

**Theorem: Vertical angles are congruent.**

**Given:**  $\angle AOB$  and  $\angle COD$  are straight angles

**Prove:**  $\angle 1 \cong \angle 3$

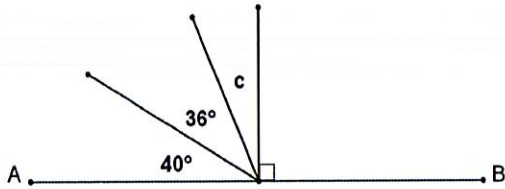


Statements	Reasons
① $\angle AOB$ and $\angle COD$ are straight angles	① Given
② $\angle 1$ and $\angle 2$ are a linear pair $\angle 2$ and $\angle 3$ are a linear pair	② They are both pairs of adjacent, supplementary angles (def. of linear pair)
③ $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	③ If two angles form a linear pair, then they are supplementary
④ $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	④ If two angles are supplementary, then they sum to $180^\circ$ (def. of supplementary)
⑤ $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	⑤ Substitution prop.
⑥ $m\angle 1 = m\angle 3$	⑥ Subtraction prop.
⑦ $\angle 1 \cong \angle 3$	⑦ Angles that have equal measure are congruent (def. of congruent angles)

FINAL EXAM REVIEW – Angle Relationships and Angle Proofs

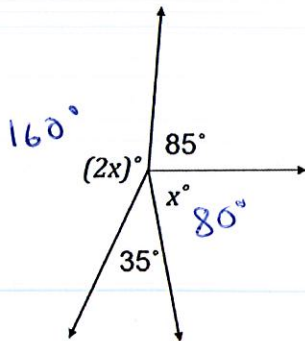
Directions: Find the value of each "lettered" angle. Explain your step-by-step process. For example: Angle  $b$  is 50 degrees because it is part of a linear pair.

1.



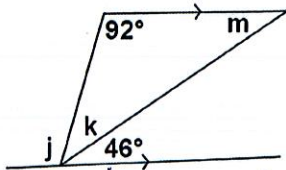
- consecutive, adjacent angles on a line sum to  $180^\circ$
- $90 + c + 36 + 40 = 180$   
 $c = 14^\circ$

2.



- angles at a point sum to  $360^\circ$
- $2x + 85 + x + 35 = 360$   
 $x = 80^\circ$ ,  $2x = 160^\circ$

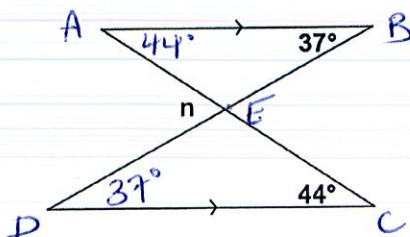
3.



\* multiple ways to solve

- ①  $m = 46^\circ$  b/c alt. int. angles are congruent when lines are parallel
- ②  $j = 92^\circ$  (same reason at #1)
- ③  $k = 42^\circ$  b/c consecutive, adjacent angles on a line sum to  $180^\circ$

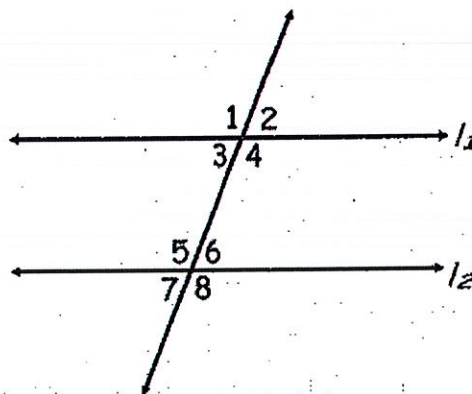
4.



- ①  $m\angle D = 37^\circ$ ,  $m\angle A = 44^\circ$  b/c alt. int. angles are congruent when lines are parallel
- ②  $m\angle CED = 99^\circ$  b/c the interior angles of a  $\Delta$  sum to  $180^\circ$
- ③  $n = 81^\circ$  b/c linear pairs sum to  $180^\circ$

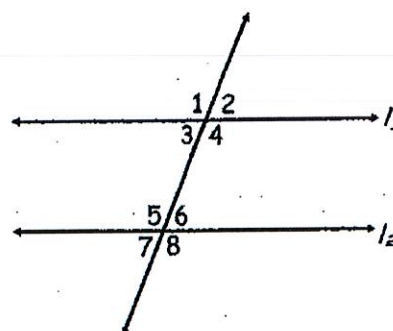


Given:  $l_1 \parallel l_2$   
Prove:  $\angle 3 \cong \angle 6$



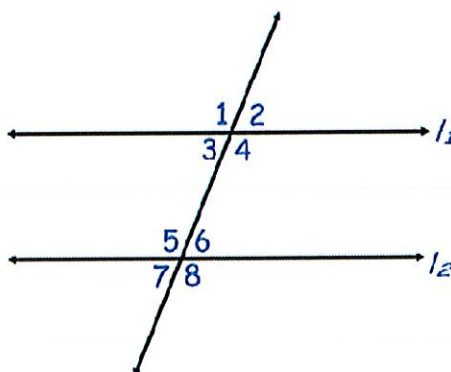
✖

Given:  $l_1 \parallel l_2$   
Prove:  $\angle 1$  is supplementary  
to  $\angle 7$



Given:  $\angle 3 \cong \angle 6$

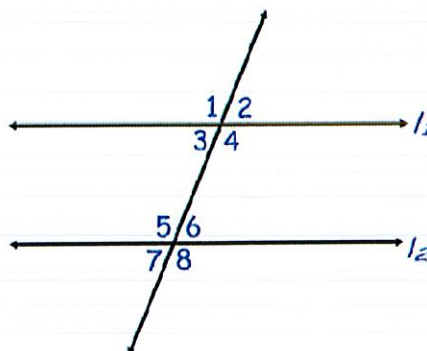
Prove:  $l_1 \parallel l_2$



✖

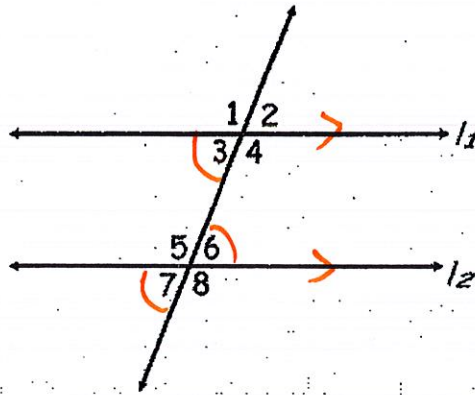
Given:  $\angle 3$  and  $\angle 5$  are  
supplementary

Prove:  $l_1 \parallel l_2$



Given:  $l_1 \parallel l_2$

Prove:  $\angle 3 \cong \angle 6$



Statements

Reasons

①  $l_1 \parallel l_2$

②  $\angle 3 \cong \angle 7$

③  $\angle 7 \cong \angle 6$

④  $\angle 3 \cong \angle 6$

① Given

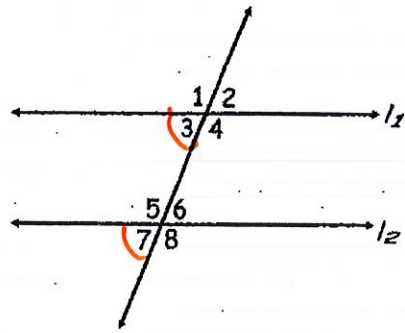
② corresponding angles are congruent when lines are parallel

③ vertical angles are  $\cong$

④ transitive property

( $\angle 3$  and  $\angle 6$  are both congruent to  $\angle 7$ , so they are congruent to each other)

Given:  $l_1 \parallel l_2$   
 Prove:  $\angle 1$  is supplementary  
 to  $\angle 7$

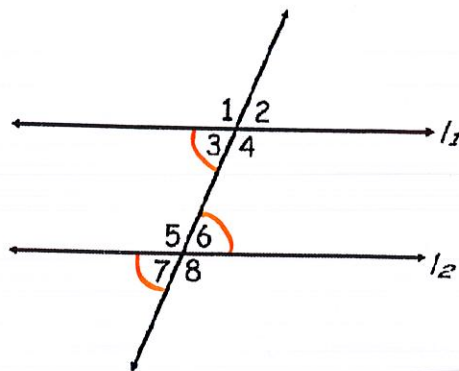


Statements	Reasons
① $l_1 \parallel l_2$	① Given
② $\angle 3 \cong \angle 7$ ( $m\angle 3 = m\angle 7$ )	② corresponding angles are $\cong$ when lines are parallel
③ $m\angle 1 + m\angle 3 = 180^\circ$	③ linear pairs are supplementary
④ $m\angle 1 + m\angle 7 = 180^\circ$	④ substitution ( $\angle 7$ is substituted for $\angle 3$ )
⑤ $\angle 1$ is supplementary to $\angle 7$	⑤ definition of supplementary (b/c they sum to $180^\circ$ )



Given:  $\angle 3 \cong \angle 6$

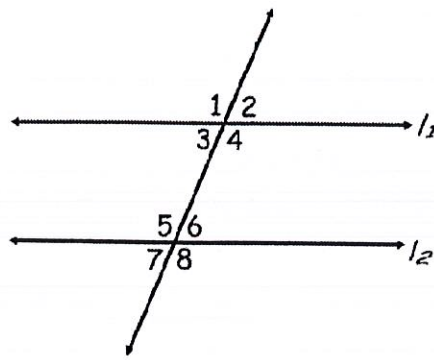
Prove:  $l_1 \parallel l_2$



Statements	Reasons
① $\angle 3 \cong \angle 6$	① Given
② $\angle 6 \cong \angle 7$	② vertical angles are $\cong$
③ $\angle 3 \cong \angle 7$	③ transitive property
④ $l_1 \parallel l_2$	④ If corresponding angles are congruent ( $\angle 3 \cong \angle 7$ ), then lines are parallel.

Given:  $\angle 3$  and  $\angle 5$  are supplementary

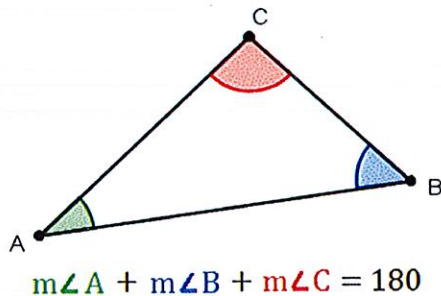
Prove:  $l_1 \parallel l_2$



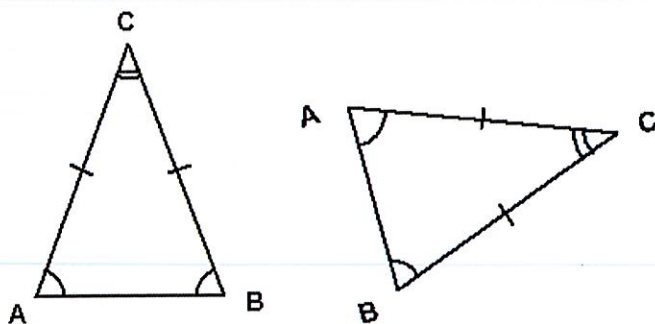
Statements	Reasons
① $\angle 3$ and $\angle 5$ are supplementary	① Given
② $m\angle 3 + m\angle 5 = 180^\circ$	② definition of supplementary (they sum to $180^\circ$ )
③ $m\angle 3 + m\angle 1 = 180^\circ$	③ linear pairs are supplementary
④ $m\angle 3 + m\angle 1 = m\angle 3 + m\angle 5$	④ substitution
⑤ $m\angle 1 = m\angle 5$ ( $\angle 1 \cong \angle 5$ )	⑤ subtraction
⑥ $l_1 \parallel l_2$	⑥ when corresponding angles are congruent, lines are parallel

# FINAL EXAM REVIEW – Properties of Triangles

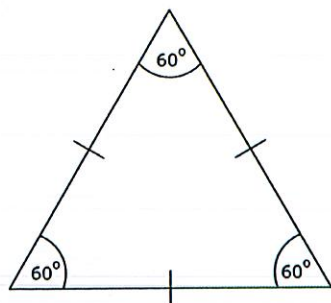
1. The sum of the interior angles of any triangle is 180 degrees.



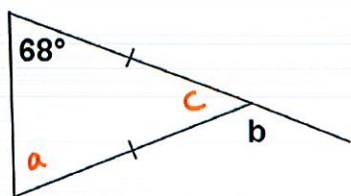
2. The base angles of an isosceles triangle are congruent to each other.



3. All interior angles of an equilateral triangle are 60 degrees.



Ex:

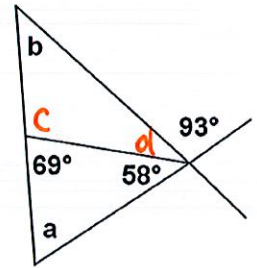


$a = 68^\circ$  b/c base angles of an isosceles  $\Delta$  are  $\cong$   
 $c = 44^\circ$  b/c the int. angles of a  $\Delta$  sum to  $180^\circ$   
 $b = 136^\circ$  b/c linear pairs sum to  $180^\circ$



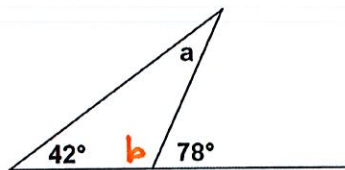
1. Find the measures of angles  $a$  and  $b$  in the figure to the right. Justify your results.

- ①  $d = 29^\circ$  b/c consecutive adjacent angles on a line sum to  $180^\circ$   
 ②  $a = 53^\circ$  b/c int. angles of a  $\Delta$  sum to  $180^\circ$  "  
 ③  $b = 40^\circ$  b/c "



In each figure, determine the measures of the unknown (labeled) angles. Give reasons for your calculations.

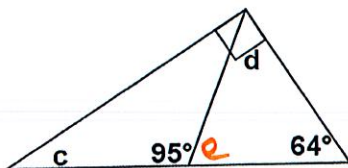
2.



$$m\angle a = \underline{36^\circ}$$

- $b = 102^\circ$  b/c linear pairs are supplementary  
 $a = 36^\circ$  b/c int. angles of a  $\Delta$  sum to  $180^\circ$

3.



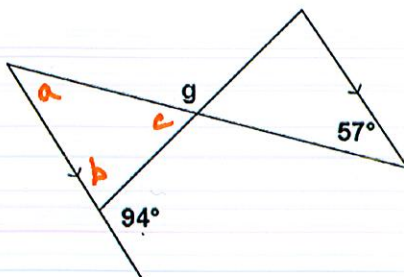
$$c = 26^\circ \text{ b/c int. angles of a } \Delta \text{ sum to } 180^\circ$$



$$e = 85^\circ \text{ b/c linear pairs sum to } 180^\circ$$

$$d = 31^\circ \text{ b/c int. angles of a } \Delta \text{ sum to } 180^\circ$$

4.



$$a = 57^\circ \text{ b/c alt. int. angles are } \cong \text{ when lines are parallel}$$

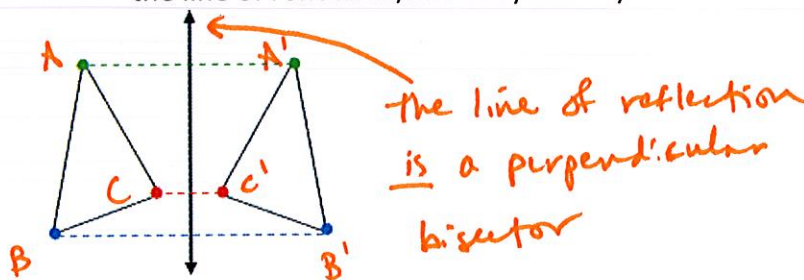
$$b = 86^\circ \text{ b/c linear pairs sum to } 180^\circ$$

$$c = 37^\circ \text{ b/c int. angles of a } \Delta \text{ sum to } 180^\circ$$

$$g = 143^\circ \text{ b/c linear pairs sum to } 180^\circ$$

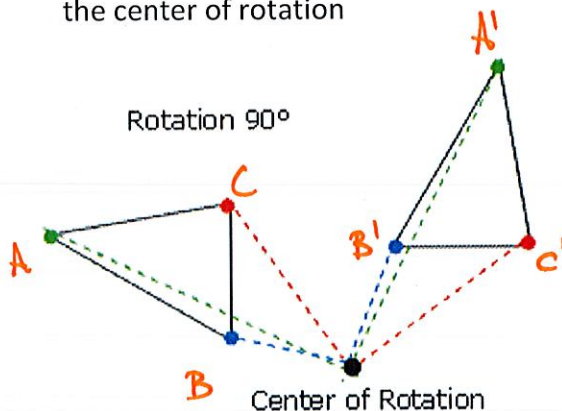
# FINAL EXAM REVIEW – Transformations

- A **transformation** is a **function**: There is a set of points (the pre-image), which is the input. We apply a rule (for example, translate right 4 and up 6) and then there is a new location for the set of points (the image), which is the output.
- There are three rigid motions: rotations, reflections, translations
- A **rigid motion** maps the pre-image onto the image. It preserves angle measurements and distances; therefore, the image and pre-image are **congruent** after a series of rigid motions.
- **Reflection**: corresponding points of the pre-image and image are equal distances from the line of reflection/line of symmetry

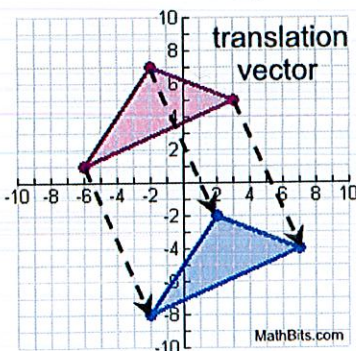


All points on a perpendicular bisector are equidistant from the endpoints of the segment being bisected.

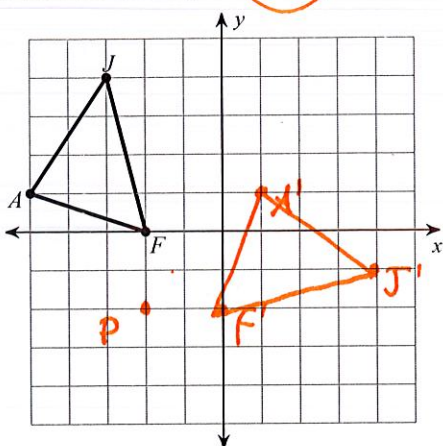
- **Rotation**: corresponding points of the pre-image and image are equal distances from the center of rotation



- **Translation**: corresponding points of the pre-image and image are equal distances from each other along the same vector



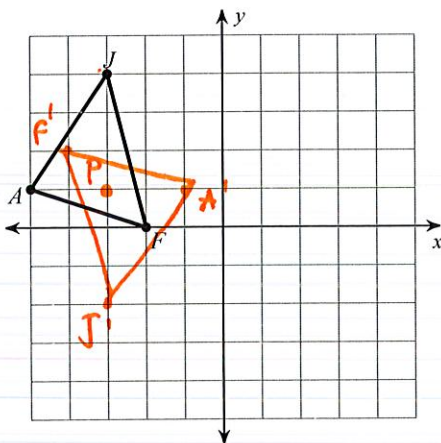
1. Rotate the figure  $-90^\circ$  about the center  $P(-2, -2)$ . Label the resulting image appropriately.



Indicate which statements are true:

- ☒ A.  $\triangle AJF$  and  $\triangle A'J'F'$  have the same perimeter
- ☒ B.  $\triangle AJF$  and  $\triangle A'J'F'$  have the same area
- ☒ C.  $m\angle A = m\angle A'$
- ☐ D. F and F' have the same location
- ☒ E.  $JF = J'F'$
- ☐ F.  $FF' = JJ'$
- ☒ G.  $FP = F'P$

2a. Rotate the figure  $180^\circ$  about the center  $P(-3, 1)$ . Label the resulting image appropriately.



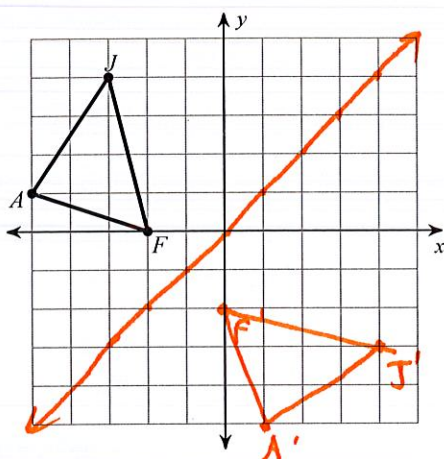
2b. What is the measure of  $\angle FPF'$ ?

$180^\circ$

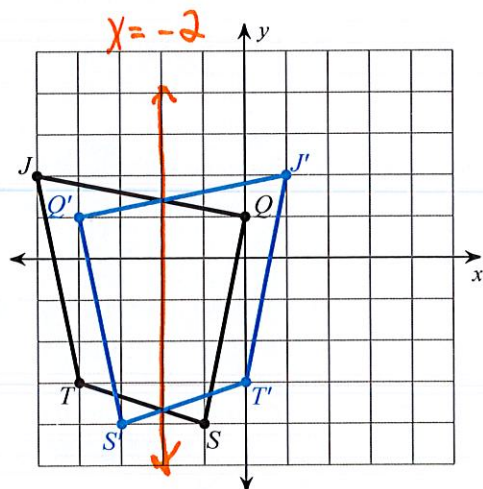


$$y = x + 0 \rightarrow \text{slope is } 1, \text{ y-intercept is } 0$$

3. Reflect the figure across  $y = x$ . Label the image appropriately. (3 pts.)



4. There is a reflection that transforms JQST to J'Q'S'T'. Draw the line of reflection and write the equation of the line. Then, describe **why** that is the line of reflection.



Line of reflection:  $x = -2$

This is the line of reflection b/c all corresponding points from the pre-image and image are equidistant from it.

5. Describe how a transformation is a function.

A transformation is a function because there is an input (a set of points - pre-image) then you apply a rule (reflect, rotate, etc.) to produce an output (image).

6. Name three rigid motions and describe how they are related to congruence?

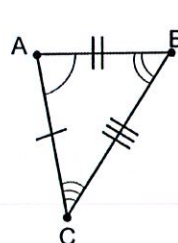
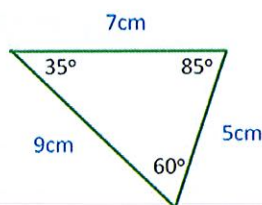
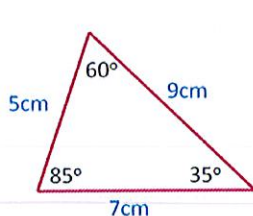
① Rotation, ② Reflection, ③ translation These transformations maintain congruence between the pre-image and the image.

7. Describe how a perpendicular bisector relates to the line of symmetry in a reflection. Based on what you know about perpendicular bisectors, how do you know all corresponding points are equidistant from the line of symmetry?

The perpendicular bisector of corresponding point on the pre-image and image is the line of symmetry. All corresponding points are endpoints of segments that are bisected so they are equidistant from the perpendicular bisector/line of symmetry.

# FINAL EXAM REVIEW – Triangle Proofs

- Two triangles are congruent if all **corresponding sides** are the **same length**, and if all **corresponding angles** have the **same measure**.
- Both pairs of triangles below are congruent. You can tell the first pair is congruent because corresponding sides and angles are the same measure. You can tell the second pair is congruent because of the congruence markings.



- Shortcuts for proving triangles congruent:

Congruence	Explanation	Diagram
SSS	When two triangles have three corresponding sets of sides congruent, use SSS to say the triangles are congruent.	
SAS	When two triangles have two pairs of sides congruent and the angles between them are congruent, use SAS to say the triangles are congruent.	
ASA	When two triangles have two pairs of angles congruent and the sides between them are congruent, use ASA to say the triangles are congruent.	
AAS	When two triangles have two pairs of angles congruent and the sides not between them are congruent, use AAS to say the triangles are congruent.	
HL	When two right triangles have congruent hypotenuses and a pair of congruent legs, use HL to say the triangles are congruent.	

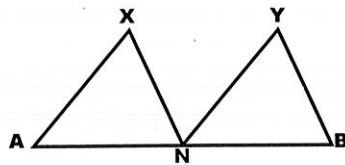
**CAREFUL! AAA** (Angle Angle Angle) and **SSA** (Side Side Angle) do **NOT** prove two triangles congruent! DO NOT USE THEM IN A PROOF FOR TRIANGLE CONGRUENCE!



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**Geometry**

- Important Concepts for Proofs

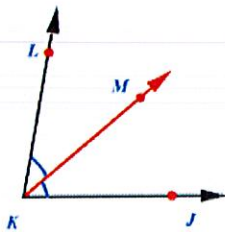
- Midpoint: the middle point of a line segment; It is equidistant from both endpoints; it bisects the segment.



In the diagram to the left:

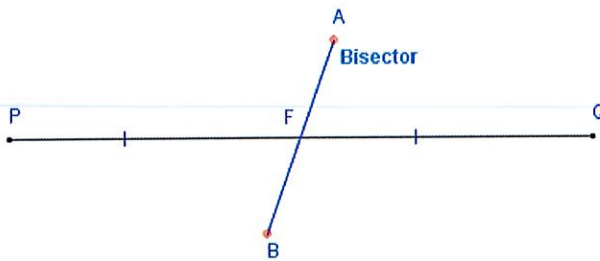
**$N$  is the midpoint of  $\overline{AB}$  so...**  
 **$\overline{AN} \cong \overline{BN}$**

- Bisector: a line that cuts an angle or line segment into two equal parts



In the diagram to the left:

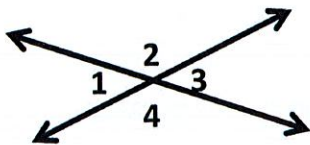
**$\overrightarrow{KM}$  bisects  $\angle LKJ$  so...  $\angle LKM \cong \angle JKM$**



In the diagram to the left:

**$\overline{AB}$  bisects  $\overline{PQ}$  so...  $\overline{PF} \cong \overline{QF}$**

- Vertical angles



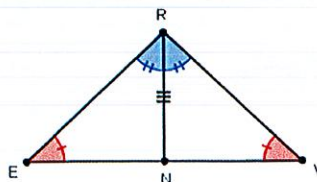
In the diagram to the left:

**Angles 1 and 3 are vertical angles so they are CONGRUENT.**

**Angles 2 and 4 are vertical angles so they are CONGRUENT.**

- Corresponding Parts of Congruent Triangles are Congruent (CPCTC): If two triangles are congruent, then the corresponding parts of those triangles are congruent!

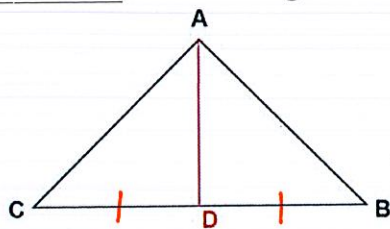
- Reflexive Property – in the diagram below  $\overline{RN} \cong \overline{RN}$  by the reflexive property





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Directions: Use the diagram to answer the questions below.



$\overline{AD}$  bisects  $\overline{CB}$ ; therefore,  $\overline{CD} \cong \overline{BD}$ . You are asked to prove  $\triangle ADC \cong \triangle ADB$ .

a. To prove congruence by SSS, what two additional congruence statements are needed?

1.  $\overline{AC} \cong \overline{AB}$

2.  $(\overline{AD} \cong \overline{AD}) \rightarrow \text{implied from the diagram}$

b. To prove congruence by SAS, what two additional congruence statements are needed?

1.  $\angle ADC \cong \angle ADB$

2.  $(\overline{AD} \cong \overline{AD}) \rightarrow \text{implied from the diagram}$

c. To prove congruence by ASA, what two additional congruence statements are needed?

1.  $\angle ADC \cong \angle ADB$

2.  $\angle ACD \cong \angle ABD$   
 $(\angle C \cong \angle B)$

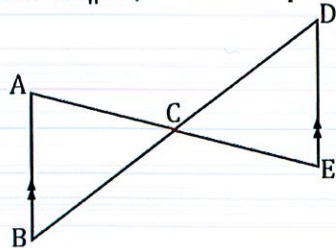
d. To prove congruence by AAS, what two additional congruence statements are needed?

1.  $\angle BAD \cong \angle CAD$

2.  $\angle ADC \cong \angle ADB$

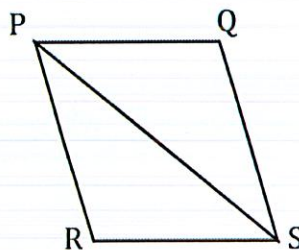
COMPLETE ON A SEPARATE SHEET OF PAPER

Given:  $\overline{AB} \parallel \overline{DE}$ , C is the midpoint of  $\overline{AE}$

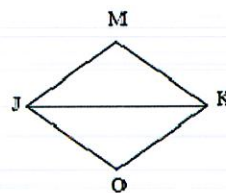


Prove:  $\overline{BC} \cong \overline{DE}$

Given: PQRS is a parallelogram



Prove:  $\triangle RPS \cong \triangle QSP$

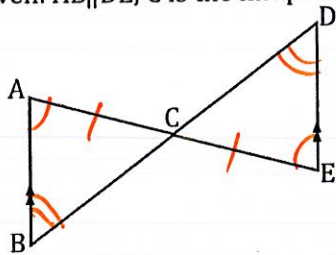


Given:  $\overline{MK} \cong \overline{OK}$

$\overline{KJ}$  bisects  $\angle MKO$

Prove:  $\overline{KJ}$  bisects  $\angle MJO$

Given:  $\overline{AB} \parallel \overline{DE}$ , C is the midpoint of  $\overline{AE}$



Prove:  $\overline{BC} \cong \overline{DC}$

Statements

Reasons

①  $\overline{AE} \parallel \overline{DE}$ ,  
C is the midpt. of  $\overline{AE}$

① Given

②  $\overline{AC} \cong \overline{EC}$

② definition of a  
midpt. (a midpt.  
divides a segment  
into 2  $\cong$  parts)

③  $\angle CAB \cong \angle DEC$ ,  
 $\angle ABC \cong \angle EDC$

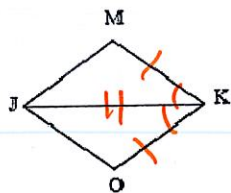
③ alt. int. angles  
are  $\cong$  when lines  
are parallel

④  $\triangle ACB \cong \triangle EDC$

④ AAS

⑤  $\overline{BC} \cong \overline{DC}$

⑤ CPCTC



Given:  $\overline{MK} \cong \overline{OK}$

$\overline{KJ}$  bisects  $\angle MKO$

Prove:  $\overline{KJ}$  bisects  $\angle MJO$

Statements

Reasons

①  $\overline{MK} \cong \overline{OK}$ ,  
 $\overline{KJ}$  bisects  $\angle MKO$

① Given

②  $\angle MKJ \cong \angle OKJ$

② def. of bisect  
(angle bisector cuts  
an angle into 2  $\cong$   
parts)

③  $\overline{JK} \cong \overline{JK}$

③ reflexive property

④  $\triangle MKJ \cong \triangle OKJ$

④ SAS

⑤  $\angle MJK \cong \angle OJK$

⑤ CPCTC

⑥  $\overline{KJ}$  bisects  $\angle MJO$

⑥ def. of bisect

## Statements

- ① PQRS is a parallelogram
- ②  $\overline{PQ} \parallel \overline{SR}$ ,  
 $\overline{QS} \parallel \overline{RP}$
- ③  $\angle QPS \cong \angle RSP$ ,  
 $\angle RPS \cong \angle QSP$
- ④  $\overline{PS} \cong \overline{PS}$
- ⑤  $\triangle RPS \cong \triangle QSP$

## Reasons

- ① Given
- ② def. of p-gram
- ③ alt. int. angles are congruent when lines are parallel
- ④ reflexive property
- ⑤ ASA



FINAL EXAM REVIEW – Properties of Parallelograms

**Parallelogram**

1. Both pairs of opposite sides are parallel.
2. Both pairs of opposite sides are congruent.
3. One pair of opposite sides are parallel and congruent.
4. Diagonals bisect each other.
5. Both pairs of opposite angles are congruent.
6. Consecutive angles are supplementary.

**Rectangle**

1. All the properties of a parallelogram.
2. Has a right angle.
3. Diagonals are congruent.

**Rhombus**

1. All the properties of a parallelogram.
2. All sides are congruent.
3. Diagonals are perpendicular.
4. Diagonals bisect the opposite angles.

**Square**

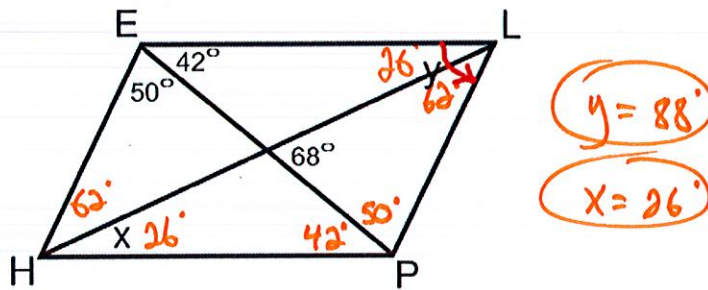
1. All the properties of a parallelogram.
2. All the properties of a rectangle.
3. All the properties of a rhombus.

definition of a parallelogram: quadrilateral with opposite sides that are parallel

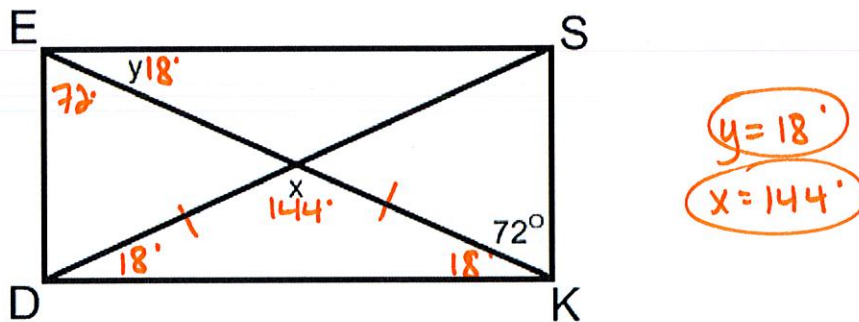
WLPCS

Geometry

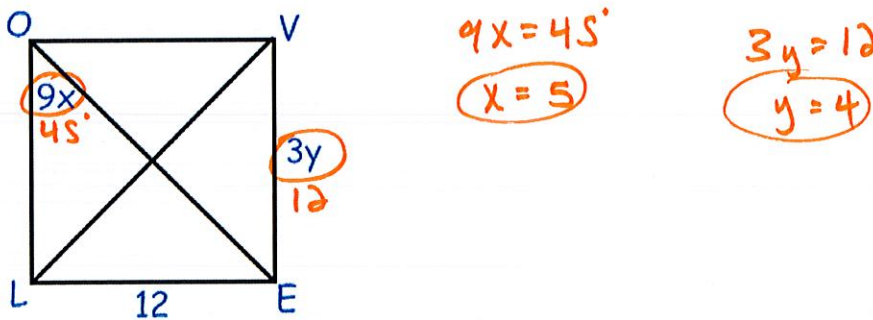
1. HELP is a parallelogram. Find the values of  $x$  and  $y$ :



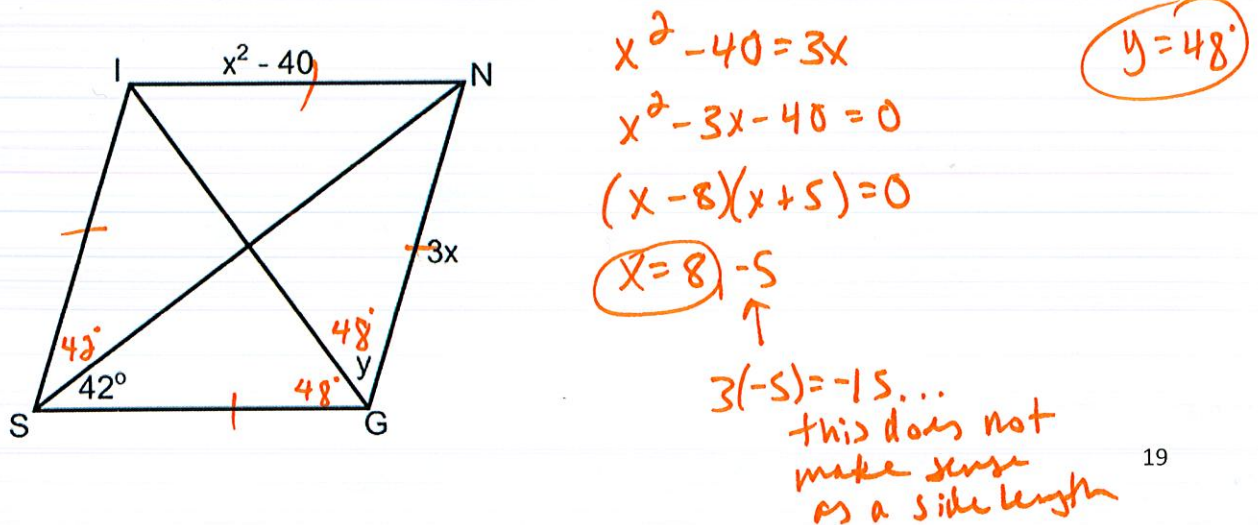
2. DESK is a rectangle. Find the values of  $x$  and  $y$ :



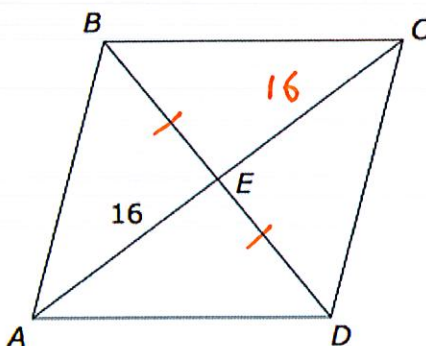
3. LOVE is a square. Find the values of  $x$  and  $y$ :



4. SING is a rhombus. Find the values of  $x$  and  $y$ :



The figure shows parallelogram  $ABCD$  with  $AE = 16$ .



not drawn to scale

**14. Part A**

Let  $BE = x^2 - 48$  and let  $DE = 2x$ . What are the lengths of  $\overline{BE}$  and  $\overline{DE}$ ? Justify your answer.

Enter your answer and your justification in the space provided.

$$\begin{aligned} x^2 - 48 &= 2x & BE &= (-6)^2 - 48 \\ x^2 - 2x - 48 &= 0 & &= -12 \text{ No!} \\ (x+6)(x-8) &= 0 & BE &= (8)^2 - 48 \\ x &= -6, 8 & & \boxed{BE = 16 = DE} \end{aligned}$$

**Part B**

What conclusion can be made regarding the specific classification of parallelogram  $ABCD$ ? Justify your answer.

Enter your answer and your justification in the space provided.

Because the diagonals are  $\cong (32 \neq 32)$ ,  
the p-gram is a rectangle.